International Journal on Emerging Technologies 10(4): 184-190(2019)

ISSN No. (Print): 0975-8364
ISSN No. (Online): 2249-3255

Sensitivity Analysis and Evaluation of Critical Size of Reactor Using Response Surface Methodology

Olatomide Gbenga Fadodun[1], Ayodeji Olalekan Salau[2], Adebimpe Amos Amosun[1] and Francis Idowu Ibitoye[1]

[1]Junior Research Fellow, Centre for Energy Research and Development, Obafemi Awolowo University Nigeria.
[2]Department of Electrical/Electronics and Computer Engineering, Afe Babalola University, Ado-Ekiti, Nigeria.
[1]Junior Research Fellow, Centre for Energy Research and Development, Obafemi Awolowo University Nigeria.
[1]Associate Professor, Centre for Energy Research and Development, Obafemi Awolowo University Nigeria.

(Corresponding author: Olatomide Gbenga Fadodun)
(Received 30 August 2019, Revised 16 October 2019, Accepted 26 October 2019)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: This work developed a Matlab app called TOWAN-2GROUP based on the two-group diffusion theory of spherical coordinates used to evaluate the critical radius and flux profile in a spherical reflected reactor using nuclear data from natural uranium/ graphite system as a case study. In addition, response surface methodology (RSM) was used to investigate the effect of four nuclear parameters in the core (diffusion length, age-thermal, fast diffusion coefficient, and thermal diffusion coefficient) of the critical radius. The analysis was carried out using ANOVA. The ANOVA results show that all the four parameters and some interaction between them are statistically significant. Furthermore, the sensitivity analysis carried out on the regression model showed that diffusion length is the most sensitive parameter of the critical radius amongst the four parameters.

Keyword: Diffusion equation, neutron flux, critical radius, response surface methodology, fast flux, thermal flux.

Abbreviations: RSM, Response surface methodology; GAEM, Generalization of the analytical exponential model; DOE, design of experiment.

I. INTRODUCTION

Criticality evaluation is essential in the design and operation of a nuclear reactor as it signifies the stable condition of the reactor. This condition is expressed by an effective multiplication factor (k_{eff}) which is the ratio of the number of neutrons in the reactor in one generation to the number of neutrons in the preceding generation [1- 3]. The process of obtaining the multiplication factor (k_{eff}) involves splitting energy-dependent diffusion equations into a finite number of groups to obtain multi-group diffusion equations which are second order coupled differential equations; It should be noted that in a practical reactor design, numerical approaches are usually employed as analytical solutions are non-existent for both complex geometry and large groups. However, for few groups and simple geometries, analytical solutions are applicable. For instance, Arzhanov [4], solved two-group diffusion equations in a spherical reflected reactor to evaluate flux distribution. The diffusion equations were solved using the method of trial function and appropriate boundary conditions to obtain flux distribution throughout the system.

Nahla & Al-Ghamdi [5] solved two-group reactor kinetics with one-group delayed neutrons in a three-dimensional homogeneous reactor using Generalization of the Analytical Exponential Model (GAEM). Ceolin *et al.* [6], solved a time-dependent multi-group neutron diffusion equation in a heterogeneous slab reactor by discretizing the global domain into sub-domains which are homogeneous. The reactor kinetic equation is then solved for each domain by Taylor expansion series. Theler [7] solved multi-group neutron diffusion equations over an unstructured grid using finite element and finite volume discretization method.

Process optimization is essential for any engineering design. It involves adjusting the values of input parameters in order to optimize certain output parameters without violating the constraints placed on the system. In recent times, researchers have employed Response Surface Methodology (RSM) to optimize different processes such as material removal rate of hybrid aluminum metal matrix composite [8] and Aerobic Granular Sludge at high temperatures [9]. Although, there are limited works on the application of RSM to nuclear reactor design. Among the few, Zhang *et al.* [10] used RSM to optimize the position of control rods in a cylindrical pressurized water reactor. The positions of four control rods were taken as input variables while power peak factor (P_{max}), maximum temperature (T_{max}), and entropy production (S_{total}) were taken as response variables. The result showed that the positions of control rods have a great influence on these parameters. Furthermore, the optimized value of these control rod positions showed that the P_{max}, T_{max}, and S_{total} were reduced by 23%, 8.7%, and 16%, respectively.

Although there are a number of commercial codes available for neutronic analysis, however, these codes are not accessible to developing countries like Nigeria. Therefore, it is imperative to develop a code which can be used by both students and researchers in the area of nuclear engineering.

Thus, this work developed a graphical user interface application with Matlab that estimates the critical size of a spherical reflected reactor and flux distribution in both the core and reflector. Furthermore, RSM was used to investigate the effect of four input parameters: diffusion length, age-to-thermal, fast diffusion coefficient, and thermal diffusion coefficient on the critical size of the reactor.

II. MATERIALS AND METHODS

The steady-state multigroup neutron diffusion equation is given as

$$D_g \nabla^2 \Phi_g(r,t) - \Sigma_{ag}\Phi(r,t) - \left[\sum_{h>g}^{G} \Sigma_{(g\to h)}\right]\Phi_g(r,t) + \left[\sum_{h=1}^{g-1} \Sigma_{(h\to g)}\right]\Phi_h(r,t) + \chi_g \sum_{h=1}^{G} \nu_h \Sigma_{fh}\Phi_h(r,t) + S_g(r,t) = 0 \qquad (1)$$

where

Φ_g is the neutron flux in group g,

Σ_{ag} is the absorption cross-section in the group g,

$\sum_h^N \Sigma_{(g\to h)}$ is the group transfer cross-section from group g,

$\sum_{h=1}^{g-1} \Sigma_{(h\to g)}$ is the group transfer cross-section from other groups to group g,

χ_g is the fission yield appearing in group g,

ν_h is the average number of fission neutrons produced per fission,

$\sum_{h=1}^{n} \Sigma_{fh}$ is the total fission cross-section from other groups, and

$S_g(r, t)$ is the source term appearing in group g.

In this section, we have simplified the derivation of the equations found in [11, 12]. Dividing the energy spectrum into fast and thermal groups, the two group diffusion equations applicable to the core and reflector are written as:

$$D_{1c}\nabla^2\Phi_{1c} - \Sigma_1\Phi_{1c} + \frac{k_\infty}{p_c}\Sigma_{2c}^a\Phi_{2c} \qquad (2)$$

$$D_{2c}\nabla^2\Phi_{2c}(r) - \Sigma_{2c}\Phi_{2c}(r) + p_c\Sigma_{1c}\Phi_{1c}(r) = 0 \qquad (3)$$

$$D_{1r}\nabla^2\Phi_{1r}(r) - (\Sigma_{1r}^a + \Sigma_r^{1\to 2})\Phi_{1r}(r) = 0 \qquad (4)$$

$$D_{2r}\nabla^2\Phi_{2r}(r) - \Sigma_{2r}\Phi_{2r}(r) + \Sigma_{1r}^{1\to 2}\Phi_{1r}(r) = 0 \qquad (5)$$

By making Φ_{1c} the subject of the formula in Eqn. (2) and substituting it into Eqn. (3), we obtained a fourth order differential as given in Eqn. (32).

$$\nabla^2\nabla^2\Phi_{1c} - \frac{(L_{2c}^2+\tau_c)\nabla^2}{\tau_c L_{2c}^2}\Phi_{1c} - \frac{(k_\infty - 1)}{\tau_c L_{2c}^2}\Phi_{1c} = 0 \qquad (6)$$

Eqn. (6) can be written as:

$$(\nabla^2 + \mu^2)(\nabla^2 - \lambda^2)\Phi_{1c} = 0 \qquad (7)$$

where

$$\mu^2 = \frac{-\left(\frac{L_{2c}^2+\tau_c}{\tau_c L_{2c}^2}\right) \pm \sqrt{\left(\frac{L_{2c}^2+\tau_c}{\tau_c L_{2c}^2}\right)^2 + 4\left(\frac{k_\infty - 1}{\tau_c L_{2c}^2}\right)}}{2} \qquad (8)$$

$$\lambda^2 \frac{\left(\frac{L_{2c}^2+\tau_c}{\tau_c L_{2c}^2}\right) \pm \sqrt{\left(\frac{L_{2c}^2+\tau_c}{\tau_c L_{2c}^2}\right)^2 + 4\left(\frac{k_\infty - 1}{\tau_c L_{2c}^2}\right)}}{2} \qquad (9)$$

μ^2 and λ^2 are called the principal and alternate bulking.

Using principal bulking, the diffusion equation can be written as:

$$\nabla^2\Phi_{1c} + \mu^2\Phi_{1c} = 0 \qquad (10)$$

The general solution in spherical coordinate is given as:

$$\Phi_{1c} = \frac{A_1 \sin\mu r}{r} + \frac{C_1 \cos\mu r}{r} \qquad (11)$$

Since the flux must be finite at $r = 0$, it implies that $C_1 = 0$, thus Eqn. (11) reduces to

$$\Phi_{1c} = \frac{A_1 \sin\mu r}{r} \qquad (12)$$

Since fast and thermal flux have the same profile, they will only differ in amplitude. Thus, the thermal flux can be written as:

$$\Phi_{2c} = \frac{A_2 \sin\mu r}{r} \qquad (13)$$

The ratio of A_2 to A_1 is called the principal coupling coefficient and is given by Eqn. (14).

$$\frac{A_2}{A_1} = \frac{p_c\Sigma_{1c}}{(D_{2c}\mu^2 + \Sigma_{2c})} \equiv S_1 \qquad (14)$$

Using alternate bulking and following the same process, the general solution for fast flux becomes

$$\Phi_{1c} = \frac{A_1^* \sinh\lambda r}{r} + \frac{C_1^* \cosh\lambda r}{r} \qquad (15)$$

Since the flux must vanish at $r = \infty$ and $C_1^* = 0$, Eqn. (15) reduces to Eqn. (16).

$$\Phi_{1c} = \frac{A_1^* \sinh\lambda r}{r} \qquad (16)$$

Therefore, the thermal flux can also be written as:

$$\Phi_{2c} = \frac{A_2^* \sinh\lambda r}{r} \qquad (17)$$

Alternate couple S_2 is given as:

$$\frac{A_2^*}{A_1^*} = \frac{p_c \, {\Sigma_{1c}}/{\Sigma_{2c}}}{(1 - L_{2c}^2\lambda^2)} = S_2 \qquad (18)$$

The complete flux distributions for the two groups in the core are given as:

$$\Phi_{1c} = \frac{A \sin\mu r}{r} + \frac{C \sinh\lambda r}{r} = AX + CY \qquad (19)$$

$$\Phi_{2c} = \frac{AS_1 \sin\mu r}{r} + \frac{CS_2 \sinh\lambda r}{r} = AS_1X + CS_2Y \qquad (20)$$

Performing the same process, the flux distribution in the reflector is given as:

$$\Phi_{1r} = \frac{F \sinh(R + T + d - r)}{r} = FZ_1 \qquad (21)$$

$$\Phi_{2r} = \frac{\sinh \frac{1}{L_{2r}}(R + T + d - r)}{r}$$

$$\frac{S_3 \sinh\frac{1}{L_{1r}}(R+T+d-r)}{r} = GZ_2 + S_3FZ_1 \qquad (22)$$

where $$S_3 = \frac{{\Sigma_{1r}^{1\to 2}}/{D_{2r}}}{\frac{1}{\tau_r} - \frac{1}{L_{2r}^2}} \qquad (23)$$

A. Criticality condition

The continuity conditions for two-group reflected reactor states that, at the core-reflector interface both the flux and neutron current density are continuous [11]. This implies that

$$\Phi_{1c}(R) = \Phi_{1r}(R) \quad (24)$$
$$\Phi_{2c}(R) = \Phi_{2r}(R) \quad (25)$$
$$-D_{1c}\nabla\Phi_{1c}(R) = -D_{1r}\nabla\Phi_{1r}(R) \quad (26)$$
$$-D_{2c}\nabla\Phi_{2c}(R) = -D_{2r}\nabla\Phi_{2r}(R) \quad (27)$$

Substituting the expression of fluxes into Eqns. (24) to (27) we have:

$$AX + CY - FZ_1 = 0 \quad (28)$$
$$D_{1c}AX' + D_{1c}CY - D_{1r}FZ_1' = 0 \quad (29)$$
$$AS_1X + CS_2Y - GZ_2 - S_3FZ_1 = 0 \quad (30)$$
$$D_{2c}AS_1X' + D_{2c}CS_2Y' - D_{2r}GZ_2' - D_{2r}S_3FZ_1' = 0 \quad (31)$$

Note that the prime denotes the first derivative of the function with respect to *r*.

Eqns. (28) to (31) can be presented in matrix form as:

$$\begin{pmatrix} X & Y & -Z_1 & 0 \\ D_{1c}X' & D_{1c}Y' & -D_{1r}Z_1 & 0 \\ S_1X & S_2Y & -S_3Z_1 & -Z_2 \\ D_{2c}S_1X' & D_{2c}S_2Y' & -D_{2r}S_3Z_1' & -D_{2r}Z_2' \end{pmatrix}\begin{pmatrix} A \\ C \\ F \\ G \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (32)$$

For a non-trivial solution of Eqn. (32), the determinant of the matrix of the coefficient must be equal to zero as given in Eqn. (33).

$$\Delta = \begin{vmatrix} X & Y & -Z_1 & 0 \\ D_{1c}X' & D_{1c}Y' & -D_{1r}Z_1' & 0 \\ S_1X & S_2Y & -S_3Z_1 & -Z_2 \\ D_{2c}S_1X' & D_{2c}S_2Y' & -D_{2r}S_3Z_1' & -D_{2r}Z_2' \end{vmatrix} 0 \quad (33)$$

Eqn. (33) is called two-group critical determinant for the reflected reactor. Criticality can be achieved by either varying the concentration of the fuel or the size of the core in such a way that the critical determinant is equal to zero.

B. Evaluation of flux constants

In order to evaluate the magnitude of the fast and thermal flux, it is paramount to determine the constants *A, C, F,* and *G*. However, it is not possible to determine their values absolutely because they are linearly dependent on each other. To compute these constants, we usually evaluate the other three constants in-terms of *A* while *A* is determined by the operating power of the reactor. The procedure to obtain this is available in [12]. Their values are given in Eqns. (34), (35), and (36).

$$C = \left(\frac{AX}{\beta Y}\right)\left(D1c\frac{X'}{X} - D_{1r}\frac{Y'}{Y}\right) \quad (34)$$

$$F = \frac{AX}{\beta Z_1}\left(D_{1c}\frac{X'}{X} - D_{1r}\frac{Y'}{Y}\right) \quad (35)$$

$$G = \frac{AX}{\beta Z_2}\left[D_{1c}(S_2 - S_3)\frac{X'}{X} + D_{1c}(S_3 - S_1)\frac{Y'}{Y} + D_{1r}(S_1 - S_2)\frac{Z_1'}{Z_1}\right] \quad (36)$$

C. TOWAN-2 GROUP Code

A Matlab App called TOWAN-2GROUP was developed in this study to compute the critical radius and flux distribution in a spherical reflected reactor system. The governing equations derived in this paper have been implemented in the developed code in the following manner.

(i) The first section of the program asks the user to supply nuclear data. Afterwards, this data is used to calculate the principal and alternate bulking values using Eqns. (8) and (9).

(ii) The second section evaluates the coupling coefficients S_1, S_2, and S_3 using Eqns. (14), (18), and (24).

(iii) The third section computes the values of X, Y, Z_1, Z_2 using Eqns. (19), (20), (21), and (22) and their first derivatives with respect to *r* at the core-reflector interface.

(iv) The fourth section computes the determinant function of Eqn. (33) and plots it over a given interval specified by the user to obtain the critical radius.

(v) The last section of the program asks the user to supply the value of the critical radius and uses it to evaluate the values of coefficients (C, F, and G), and plots the flux profile for both the core and the reflector over a given interval which is supplied by the user. The major advantage of this code is that the computation time is much shorter as compared to when other iterative methods are used.

D. Response surface methodology

RSM is a set of advanced DOE techniques that makes use of mathematical and statistical approaches to establish a relationship between an input variable and an output response. The multivariate model for the output variable is given as:

$$y = \alpha_0 + \sum_{i=1}^{4}\alpha_i x_i + \sum_{i=1}^{4}\alpha_{ii} x_i x_i + \sum_{i=1}^{4}\alpha_{ij} x_i x_j \quad (37)$$

where $\alpha_0, \alpha_i, \alpha_{ii}, \rightleftarrows\rightleftarrows \alpha_{ij}$ are the intercept, linear term coefficient, the quadratic term coefficient, and coefficient of an i^{th} & j^{th} term respectively. This study employed the central composite design method to study the effect of four variables in the core, namely: diffusion length, age-thermal, fast diffusion coefficient, and thermal diffusion coefficient of the critical size of the reactor. Each factor is set to 5 levels and an alpha value of 2 base on the number of variables and levels. In addition, a condition of 30 runs was defined which consist of 16 factorial points, 8 axial points, and 6 center points [13]. Table 1 shows the nuclear data of Uranium/ Graphite system used and Table 2 shows the variables considered in RSM and the values associated with each level.

Table 1: Nuclear data of natural uranium/graphite system.

Variable	Units	Nat U/Graphite system
Infinite multiplication constant		1.1
Thermal diffusion length, core	cm	15.8
Thermal diffusion length, reflector	cm	54.4
Age-to-thermal, core	cm^2	364
Age-to-thermal, reflector	cm^2	364
Fast diffusion coefficient core	cm	1.11
Thermal diffusion coefficient core	cm	0.88
Fast diffusion coefficient reflector	cm	1.11
Thermal diffusion coefficient in the reflector	cm	0.886
Resonance escape probability		0.9
Reflector thickness	cm	150

Table 2: Variables and the value associated with level used in RSM.

	Level				
Variable	–2	–1	0	1	2
diffusion length (L_c)	12.64	14.22	15.8	17.28	18.96
Age-thermal (τ_c)	291.2	327.6	364	400.4	436.8
Fast group coefficient (D_{1c})	0.888	0.999	1.11	1.221	1.332
Thermal group coefficient (D_{2c})	0.7104	0.7992	0.888	0.9768	1.0656

III. RESULTS AND DISCUSSION

A. Identification of the roots and flux profile

Fig. 1 shows the graph of the critical determinant as a function of the radius of a spherical reflected reactor using natural uranium fuel surrounded by graphite reflector. The roots are determined by the zeroes of the function and these occur at R = 190 cm, 450 cm, 680 cm, 940 cm, etc. The first root also called the fundamental root is the critical radius of the reactor, while the other roots are due to higher-order harmonics which normally die out if the reactor is critical and can hence be neglected. Fig. 2 shows the complete normalized flux profiles of Natural Uranium/graphite system. It was observed that within the core, the magnitude of fast flux is greater. This was attributed to the high thermal absorption cross-section and low scattering cross-section of the core material. The implication of this is that more thermal neutrons were absorbed while less fast neutrons were scattered in the thermal group. In the reflector, the reverse was the case as the magnitude of thermal flux was greater than the magnitude of fast flux.

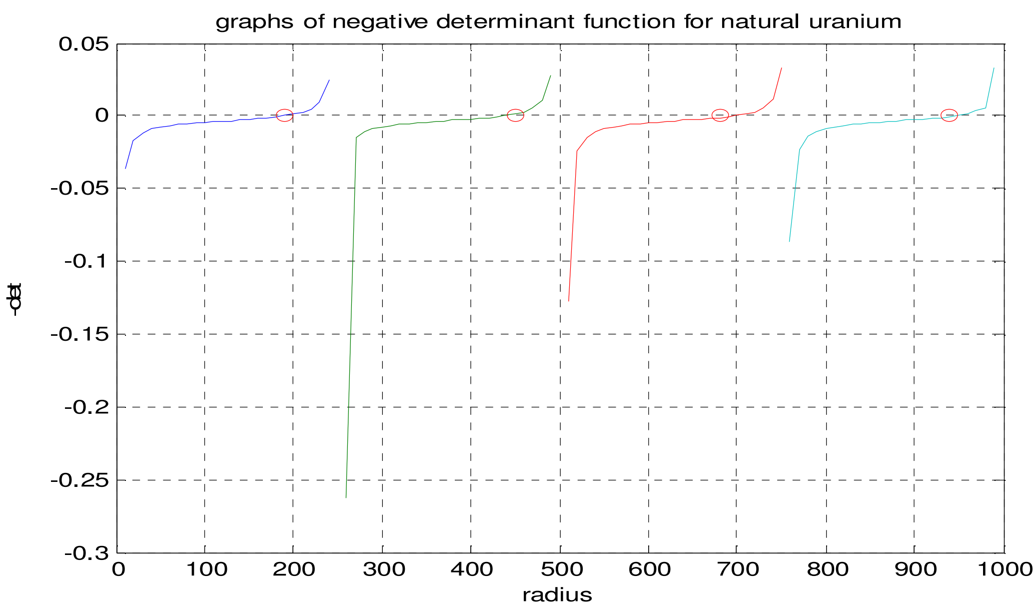

Fig. 1. Graph of negative determinant function.

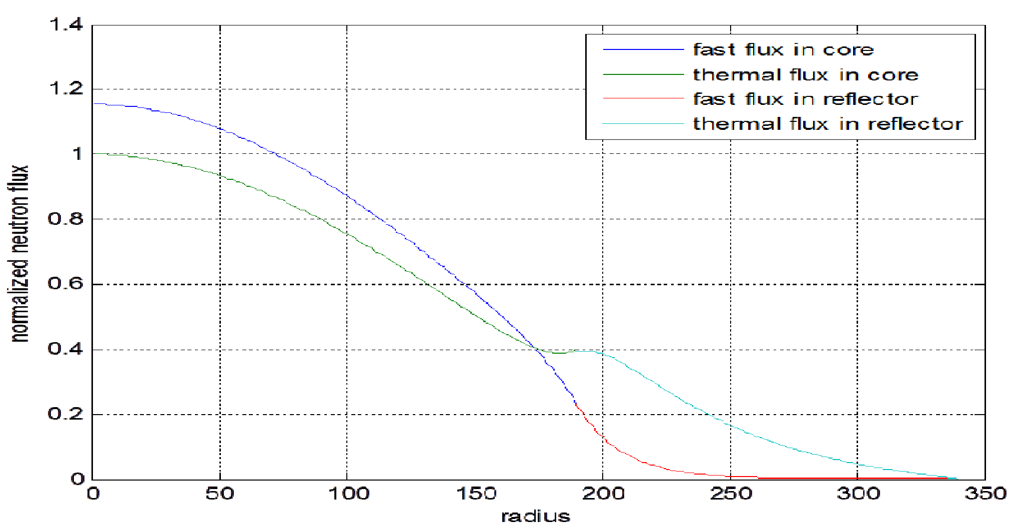

Fig. 2. Normalized flux profiles.

This change was attributed to high scattering cross-sections and low absorption cross-sections of reflector material as more fast neutrons that leak from the core were thermalized in the reflector.

B. Results using Response surface methodology

The result of the effect of four input parameters: diffusion length, age-thermal, fast diffusion coefficient, and thermal diffusion coefficient in the core on the critical size of the reactor using RSM is shown in Fig. 3. The maximum critical size obtained was 213 cm which corresponds to $(L_C = 1, \tau_c = -1, D_{1c} = -1, D_{2C} = 1)$ as shown in Table 3.

Table 4 shows the summary of the ANOVA after terms with higher P-value (terms that were not statistically significant) have been removed in order to improve the accuracy of the model. The model F- and P- values are 4563.50 and < 0.0001, respectively, which are similar to results obtained in [14]. In addition, the entire input variables $(L_C, \tau_c, D_{1c}$ and $D_{2C})$ and some interactions between them $(L_C\tau_c, L_C D_{1c}, L_C D_{2C}$ and $L_C^2)$ were all significant to the critical radius.

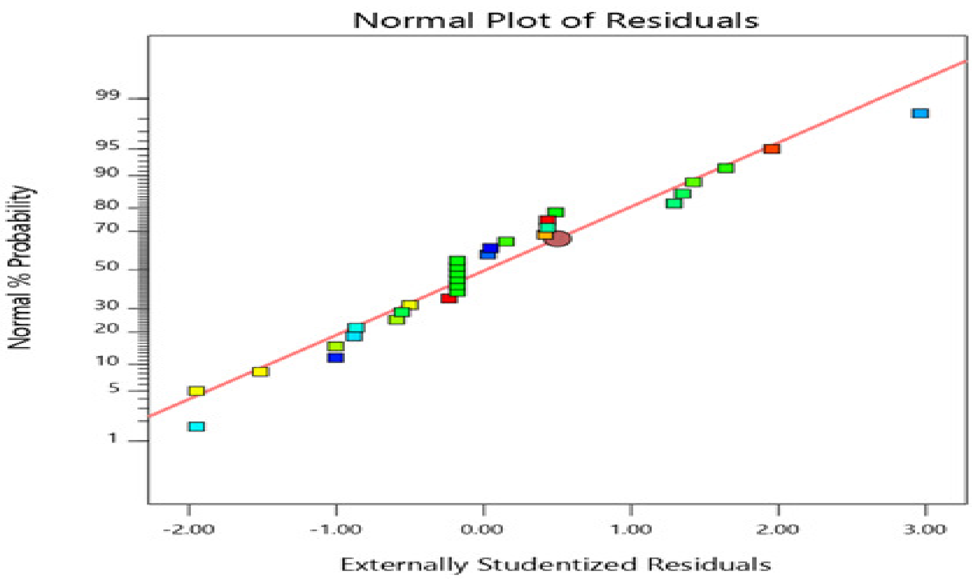

Fig. 3. Normalized plot of residual.

Table 3: The levels of the factors and the result of the critical size of the reactor.

Run	L_C	τC	D_{1C}	D_{2C}	($R_{critical}$)
1	-1	-1	-1	-1	186.4
2	1	-1	1	-1	183.2
3	-1	1	-1	-1	190
4	1	1	-1	-1	187
5	-1	-1	1	1	210
6	0	0	0	0	185
7	-2	0	0	0	193
8	-1	1	1	1	202
9	1	1	-1	1	169
10	-1	-1	-1	1	205
11	1	-1	-1	-1	190
12	0	0	0	0	201
13	1	1	1	1	175
14	0	0	-2	0	190
15	-1	1	-1	1	167.5
16	0	0	0	-2	177.5
17	0	-2	0	0	201
18	-1	-1	1	-1	178.5
19	0	0	0	0	180
20	0	0	0	0	198
21	1	-1	-1	1	213
22	2	0	0	0	189.6
23	0	0	2	0	192
24	0	0	0	2	197.5
25	1	-1	1	1	190
26	1	1	1	-1	212.7
27	0	0	0	0	190
28	0	2	0	0	172
29	0	0	0	0	190
30	-1	1	1	-1	194.4

Table 4: Result of ANOVA.

Source	Sum of Squares	df	Mean Square	F-value	p-value
Model	4180.72	9	464.52	5285.41	< 0.0001
A-diffusion length in core	2827.51	1	2827.51	32171.74	< 0.0001
B-age-thermal in core	911.43	1	911.43	10370.40	< 0.0001
C-fast diffusion coefficient in core	95.60	1	95.60	1087.75	< 0.0001
D-thermal diffusion coefficient in core	341.26	1	341.26	3882.90	< 0.0001
AB	0.6006	1	0.6006	6.83	0.0166
AC	1.05	1	1.05	11.95	0.0025
AD	1.50	1	1.50	17.07	0.0005
A²	1.29	1	1.29	14.72	0.0010
Residual	1.76	20	0.0879		
Lack of Fit	1.76	15	0.1172		
Pure Error	0.0000	5	0.0000		
Total	0.066	29			

R^2 = 0.9992, adj –R2 – pre –R2 = 0.0005, adeq Precision = 266.

Furthermore, the model R-squared value was 0.9996, and the difference between the adjusted sum of square (Adj-R^2) and predicted sum of square (Pred R^2) was 0.0005. The former indicates that 99.96% of the total variance in the critical size data can be explained by the model while the latter represents the suitability of the model as the value is less than 0.2 [15]. The regression model obtained for the critical radius is given as:

$$R_{\text{critical}}(L_C, \tau_c, D_{1c}, D_{2c}) = 190.05 + 10.85L_C + 6.16\tau_C - 2.00D_{1C} - 3.77D_{2C} - 0.194L_C\tau_c + 0.256L_cD_{1C} - 0.306L_cD_{2C} + 0.21L_C^2 \quad (38)$$

This equation is valid for $12.64 \text{ cm} \le L_C \le 18.96\text{cm}; 291.2 \text{ cm}^2 \le \tau_c \le 436.8 \text{ cm}^2;$ $0.888 \text{ cm} \le D_{1c} \le 1.33\text{cm}; 0.7104 \text{ cm} \le D_{2c} \le 1.0656 \text{ cm}.$

C. Sensitivity analysis

Sensitivity analysis is basically used to measure the effect of each variable on the output variable under certain conditions. This is achieved by taking the partial derivative of the regression models with respect to input variables. The positive value of the sensitivity connotes an increment in the output variable with respect to increment in the input variable, while a negative value implies the opposite [16, 17]. The partial derivative of the critical size with respect to the four input variables are given in Eqns. (39), (40), (41), and (42).

$$\frac{\partial R_{\text{critical}}}{\partial D_{2C}} = -3.77 - 0.306L_C \quad (39)$$

$$\frac{\partial R_{\text{critical}}}{\partial L_C} = 10.85 - 0.194\tau_c + 0.256D_{1C} - 0.306D_{2C} + 0.42L_C \quad (40)$$

$$\frac{\partial R_{\text{critical}}}{\partial \tau_c} = 6.61 - 0.194L_C \quad (41)$$

$$\frac{\partial R_{\text{critical}}}{\partial D_{1C}} = -2.00 + 0.256L_C \quad (42)$$

Table 5 shows the sensitivity of the input variables on the critical radius (R_{critical}). The results show that diffusion length (L_c) at ($L_C = 2\tau_c = -1, D_{IC} = 0, D_{2C} = 1$) has the highest sensitivity among the four parameters, followed by age-to-thermal (τ_c) at ($L_C = -2\tau_c = -1, D_{IC} = 0,\ D_{2C} = 1$), thermal diffusion coefficient (D_{2c}) at ($L_C = -2\tau_c = -1, D_{IC} = 0, D_{2C} = 1$), and lastly, fast diffusion coefficient (D_{1C}) at ($L_C = -2\tau_c = -1, D_{IC} = 0, D_{2C} = 1$).

Table 5: Sensitivity analysis of critical radius.

Variables				Sensitivity			
L	τ_c	D_{1c}	D_{2c}	L	τ_c	D_{1c}	D_{2c}
-2	-1	0	1	9.9048	6.548	-2.512	4.6836
-1	-1	0	1	10.3214	6.354	-2.256	4.3776
0	-1	0	1	10.738	6.16	-2	4.0716
1	-1	0	1	11.1546	5.966	-1.744	3.7656
2	-1	0	1	11.5712	5.772	-1.488	3.4596
1	-2	-1	0	11.3986	5.966	-1.744	3.464
1	-1	-1	0	11.2046	5.966	-1.744	3.464
1	0	-1	0	11.0106	5.966	-1.744	3.464
1	1	-1	0	10.8166	5.966	-1.744	3.464
1	2	-1	0	10.6226	5.966	-1.744	3.464
0	1	-2	-1	10.45	6.16	-2	3.4684
0	1	-1	-1	10.706	6.16	-2	3.4684
0	1	0	-1	10.962	6.16	-2	3.4684
0	1	1	-1	11.218	6.16	-2	3.4684
0	1	2	-1	11.474	6.16	-2	3.4684
-1	0	1	-2	11.3014	6.354	-2.256	3.4728
-1	0	1	-1	10.9954	6.354	-2.256	3.7744
-1	0	1	0	10.6894	6.354	-2.256	4.076
-1	0	1	1	10.3834	6.354	-2.256	4.3776
-1	0	1	2	10.0774	6.354	-2.256	4.6792

Furthermore, three of the parameters: Thermal diffusion length, age-to-thermal, and thermal diffusion coefficient have positive sensitivity while fast diffusion coefficient has negative sensitivity.

IV. CONCLUSION

In this paper, a Matlab app called TOWN-2GROUP was developed based on two group diffusion theory in a spherical coordinate to evaluate the critical radius and flux profile in a spherical reflected reactor. Response surface methodology (RSM) was also used to study the effect of four nuclear data parameters on the critical size of the reactor. The results show that all of the four input parameters and some of their interactions are statistically significant to the critical radius. Furthermore, the sensitivity analysis showed that diffusion length, age-to-thermal, and thermal diffusion coefficient have positive sensitivity while fast diffusion coefficient has negative sensitivity to the critical radius. Lastly, diffusion length was found to be the most sensitive parameter amongst the four.

V. FUTURE SCOPE

Higher group diffusion equations that are higher than two will be considered in future works as they will give a better approximation than the two–group diffusion equation which we employed in this work. Also, other reactor geometries will also be considered.

ACKNOWLEDGEMENT

Authors gratefully acknowledge the contributions of the anonymous reviewers whose valuable comments and insightful suggestions led to the improvement of the preliminary version of this paper.

Conflict of Interest. The authors declare no conflict of interest associated with this work.

REFERENCES

[1]. Urbatsch, J. T. (1995). Iterative acceleration methods for monte carlo and deterministic criticality calculations. California: Los Alamos National Laboratory, 1-174.

[2]. Ojo, B. M., Fasasi, M. K., Salau, A. O., Olukotun, S. F., & Jayeola, M. A. (2018). Criticality calculation of a homogenous cylindrical nuclear reactor core using four-group diffusion equations. *Turkish Journal of Engineering*, *2*(3), 130-138. DOI: 10.31127/tuje.411549

[3]. Jayeola, M. A., Fasasi, M. K., Amosun, A. A., Salau, A. O., & Ojo, B. M. (2018). Numerical computation of fission-product poisoning build-up and burn-up rate in a finite cylindrical nuclear reactor core. *Bilge International Journal of Science and Technology Research*, *2*(1), 17-30. DOI: 10.30516/bilgesci.397197

[4]. Arzhanov, A. (2010). Analytical models of critical reactors in simple geometries.

[5]. Nahla, A. A., & Al-Ghamdi, M. F. (2012). Generalization of the analytical exponential model for homogeneous reactor kinetics equations. *Journal of Applied Mathematics*. DOI: 10.1155/2012/282367.

[6]. Ceolin, C., Schramm, M., Vilhena, M. T., & Bodmann, B. E. (2015). On the Neutron multi-group kinetic diffusion equation in a heterogeneous slab: An exact solution on a finite set of discrete points. *Annals of Nuclear Energy*, *76*, 271-282.

[7]. Theler, G. (2013). Unstructured grids and the multigroup neutron diffusion equation. *Science and Technology of Nuclear Installations*.

[8]. Kumar, A., Singh, H. & Garg, H. K. (2016). Optimization of material removal rate (MRR) and surface roughness (SR) while turning of hybrid aluminum metal matrix composite on CNC lathe using response surface methodology (RSM). *International Journal on Emerging Technologies, 7*(1), 117-125.

[9]. Ibrahim, S., Wahab, N. A., Anuar, A. N., & Bob, M. (2017). Parameter optimisation of aerobic granular sludge at high temperature using response surface methodology. *International Journal of Electrical & Computer Engineering 7*(3), 1522-1529.

[10]. Zhang, Y. N., Zhang, H. C., Yu, H. Y., & Ma, C. (2017). Response surface methodology control rod position optimization of a pressurized water reactor core considering both high safety and low energy dissipation. *Entropy*, *19*(2), 63. DOI: 10.3390/e19020063.

[11]. Lamarsh, J. R., & Baratta, A. J. (2001). *Introduction to nuclear engineering* (Vol. 3). Upper Saddle River, NJ: Prentice hall. U.S.A.

[12]. Amosun, A. A., Salau, A. O., Fadodun, O. G., Jayeola, M. A., Osanyin, T. K., Fasasi, M. K., & Ibitoye, F. I. (2019). Numerical calculation of fuel burn-up rate in a cylindrical nuclear reactor. *Journal of Radioanalytical and Nuclear Chemistry*, *319*(1), 459-470. DOI: 10.1007/s10967-018-6361-8

[13]. Carley, K. M. Kamneva, N. Y. & Reminga, J. (2004). Response surface methodology. Carnegie-Mellon Univ. Pittsburgh Pa School of *Computer Science,* 1-26.

[14]. Shari, K. (2013). How to get started with Design Expert Software, Stat-Ease Inc.

[15]. Anderson, M. J. (2004). Response surface methods for peak process performance, 1-6, https://www.statease.com/pubs/rubplast3_rsm.pdf.

[16]. Campolongo, F., & Braddock, R. (1999). The use of graph theory in the sensitivity analysis of the model output: a second order screening method. *Reliability Engineering & System Safety*, *64*(1), 1-12.

[17]. Fadodun, O.G., Amosun, A. A., Salau, A. O., & Ogundeji, J. A. (2019). Numerical investigation of thermal performance of single-walled carbon nanotube nanofluid under turbulent flow conditions. *Engineering Reports*, Wiley. DOI: 10.1002/eng2.12024.

How to cite this article: Fadodun, Olatomide Gbenga, Salau, Ayodeji Olalekan, Amosun, Adebimpe Amos and Ibitoye, Francis Idowu (2019). Sensitivity Analysis and Evaluation of Critical Size of Reactor Using Response Surface Methodology. *International Journal on Emerging Technologies*, *10*(4): 184–190.